Modelling with the Logistic Operator Equation  
  
I will discuss a mathematical link between the Quantum Statistical Mechanics and the logistic growth and decay processes. It is based on an observation that a certain nonlinear operator evolution equation, which we refer to as the Logistic Operator Equation (LOE), provides an extension of the standard model of noninteracting bosons.

The classical logistic equation is frequently used in economics, e.g. as a model for exploitation of a renewable resource, a model for the demand curve, etc. LOE also seems to lend itself to macroeconomic interpretations. For instance, it may be viewed as a model for the volume fluctuations in a basket of physical commodities or tradable goods. In this interpretation the items in the basket are viewed as possibly interdependent, affecting one another via a complex pattern of interactions. The primary parameter of the LOE model is the reciprocal of “temperature" (rather than the more typically used time variable). However, as analysis shows, the LOE-governed change is not determined by the initial conditions. In fact, the actual effects of logistic heating or cooling, as determined from the LOE, may depend on a plethora of extraneous parameters.

Significant insight into the nature of this model can be gained by analyzing the exact solutions of LOE. I will discuss a fairly general set of solutions obtained for a special calibration of the model, which sets it in the number-theoretic framework. This trick, in the tradition of Julia and Bost-Connes, makes it possible for us to tap into the vast resources of classical mathematics and, in particular, to construct explicit solutions of LOE via the Dirichlet series. The theoretical results and numerical simulations obtained in this way shed light at the unique complexities of this rich and multifaceted model.